Welcome to AP Calculus !

Summer Assignment for AP Calculus AB and BC
North Gwinnett High School

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This packet is meant to help you review some of the mathematics that leads up to Calculus. Of course, all mathematics up until this point has simply been a foundation for Calculus, but these are the most important topics. Since the focus of this class must be the actual content of the AP test, this packet is meant only to be a review and not an in-depth course of study. If you find yourself weak in any of these areas, make sure to review them and strengthen your understanding before August. Please seek help ASAP if you need it.

Do I really need to know the stuff in this packet?

Yes, you do. These skills will appear on various assignments and assessments throughout the course. Make sure you have mastered them before school starts.

We are looking forward to meeting you and sharing an exciting, challenging, and rewarding experience together as we learn and prepare for the AP Calculus Exam!

- The North Gwinnett High School Calculus Team

Reference: https://calculus.flippedmath.com/

Calculus - SUMMER PACKET

NAME:____________________________

Summer + Math = (Best Summer Ever)^2

NO CALCULATOR!!!

Given $f(x) = x^2 - 2x + 5$, find the following.

1. $f(-2) =$

2. $f(x+2) =$

3. $f(x+h) =$

Use the graph $f(x)$ to answer the following.

4. $f(0) =$

$f(-1) =$

$f(2) =$

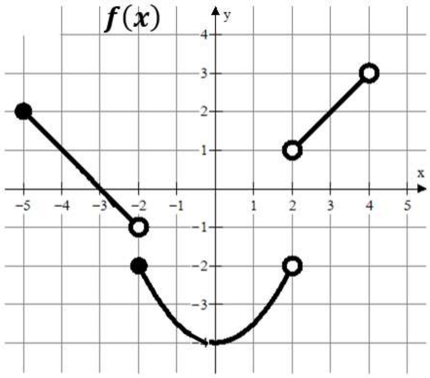
$f(x) = 2$ when x = ?

$f(4) =$

$f(-2) =$

$f(3) =$

$f(x) = -3$ when x = ?



Write the equation of the line meets the following conditions. Use point-slope form.
$y - y_1 = m(x - x_1)$

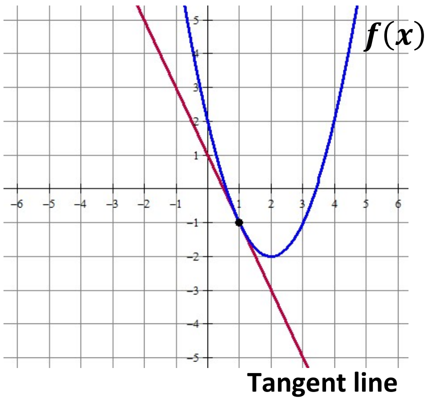
5. slope = 3 and $(4, -2)$

6. $m = -\frac{3}{2}$ and $f(-5) = 7$

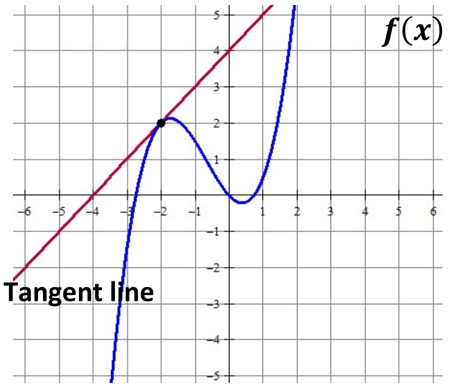
7. $f(4) = -8$ and $f(-3) = 12$

Write the equation of the tangent line in point slope form. $y - y_1 = m(x - x_1)$

8. The line tangent to $f(x)$ at $x = 1$



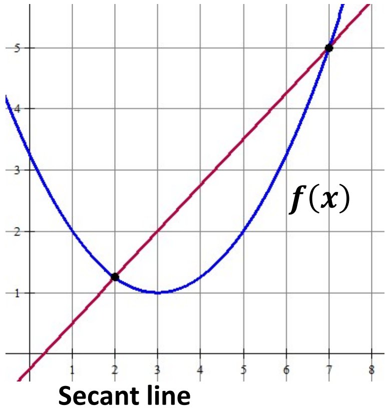
9. The line tangent to $f(x)$ at $x = -2$



MULTIPLE CHOICE! Remember slope = $\frac{y_2-y_1}{x_2-x_1}$

10. Which choice represents the slope of the secant line shown?

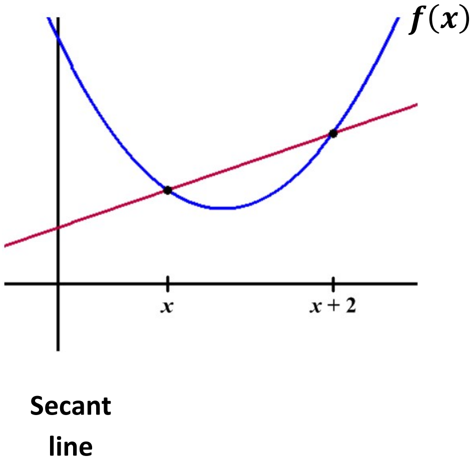
A) $\frac{7-2}{f(7)-f(2)}$ B) $\frac{f(7)-2}{7-f(2)}$ C) $\frac{7-f(2)}{f(7)-2}$ D) $\frac{f(7)-f(2)}{7-2}$



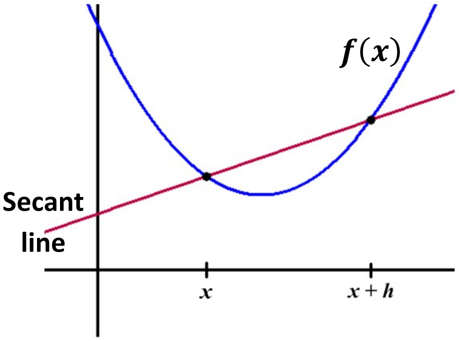
11. Which choice represents the slope of the secant line shown?

A) $\frac{f(x)-f(x+2)}{x+2-x}$ B) $\frac{f(x+2)-f(x)}{x+2-x}$ C) $\frac{f(x+2)-f(x)}{x-(x+2)}$

D) $\frac{x+2-x}{f(x)-f(x+2)}$



12. Which choice represents the slope of the secant line shown?



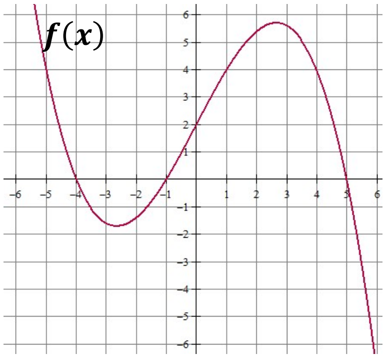
A) $\frac{f(x+h)-f(x)}{x-(x+h)}$ B) $\frac{x-(x+h)}{f(x+h)-f(x)}$ C) $\frac{f(x+h)-f(x)}{x+h-x}$

D) $\frac{f(x)-f(x+h)}{x+h-x}$

13. Which of the following statements about the function $f(x)$ is true?

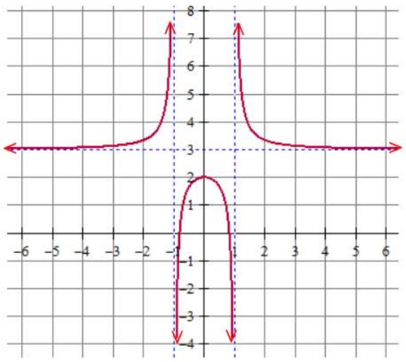
I. $f(2) = 0$
II. $(x + 4)$ is a factor of $f(x)$
III. $f(5) = f(-1)$

(A) I only
(B) II only
(C) III only
(D) I and III only
(E) II and III only



Find the domain and range (express in interval notation). Find all horizontal and vertical asymptotes.

14.



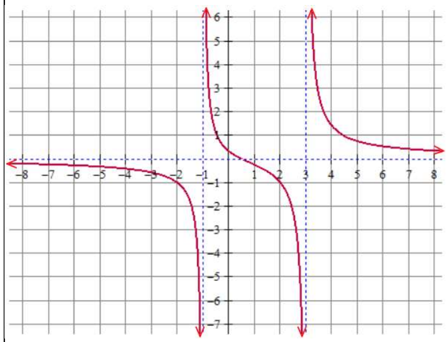
Domain:

Range:

Horizontal Asymptote(s):

Vertical Asymptotes(s):

15.



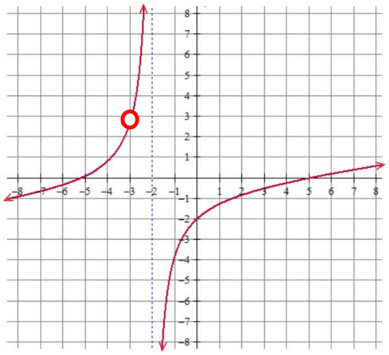
Domain:

Range:

Horizontal Asymptote(s):

Vertical Asymptotes(s):

16.



Domain:

Range:

Horizontal Asymptote(s):

Vertical Asymptotes(s):

MULTIPLE CHOICE!

17. Which of the following functions has a vertical asymptote at $x = 4$?

(A) $\frac{x+5}{x^2-4}$

(B) $\frac{x^2-16}{x-4}$

(C) $\frac{4x}{x+1}$

(D) $\frac{x+6}{x^2-7x+12}$

(E) None of the above

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18. Consider the function: $(x) = \frac{x^2-5x+6}{x^2-4}$. Which of the following statements is true?

I. $f(x)$ has a vertical asymptote of $x = 2$
II. $f(x)$ has a vertical asymptote of $x = -2$
III. $f(x)$ has a horizontal asymptote of $y = 1$

(A) I only

(B) II only

(C) I and III only

(D) II and III only

(E) I, II and III

Rewrite the following using rational exponents. Example: $\mathbf{\frac{1}{\sqrt[3]{x^2}} = x^{-\frac{2}{3}}}$

19. $\sqrt[5]{x^3} + \sqrt[5]{2x}$

20. $\sqrt{x+1}$

21. $\frac{1}{\sqrt{x+1}}$

22. $\frac{1}{\sqrt{x}} - \frac{2}{x}$

23. $\frac{1}{4x^3} + \frac{1}{2}\sqrt[4]{x^3}$

24. $\frac{1}{4\sqrt{x}} - 2\sqrt{x+1}$

Write each expression in radical form and positive exponents. Example: $\mathbf{x^{-\frac{2}{3}} + x^{-2} = \frac{1}{\sqrt[3]{x^2}} + \frac{1}{x^2}}$

25. $x^{-\frac{1}{2}} - x^{\frac{3}{2}}$

26. $\frac{1}{2}x^{-\frac{1}{2}} + x^{-1}$

27. $3x^{-\frac{1}{2}}$

28. $(x+4)^{-\frac{1}{2}}$

29. $x^{-2} + x^{\frac{1}{2}}$

30. $2x^{-2} + \frac{3}{2}x^{-1}$

Need to know basic trig functions in RADIANS! We never use degrees. You can either use the Unit Circle or Special Triangles to find the following.

31. $\sin\frac{\pi}{6}$

32. $\cos\frac{\pi}{4}$

33. $\sin 2\pi$

34. $\tan\pi$

35. $\sec\frac{\pi}{2}$

36. $\cos\frac{\pi}{6}$

37. $\sin\frac{\pi}{3}$

38. $\sin\frac{3\pi}{2}$

39. $\tan\frac{\pi}{4}$

40. $\csc\frac{\pi}{2}$

41. $\sin\pi$

42. $\cos\frac{\pi}{3}$

43. Find x where $0 \leq x \leq 2\pi$,

$$\sin x = \frac{1}{2}$$

44. Find x where $0 \leq x \leq 2\pi$,

$$\tan x = 0$$

45. Find x where $0 \leq x \leq 2\pi$,

$$\cos x = -1$$

Solve the following equations. Remember $e^0 = 1$ and $\ln 1 = 0$.

46. $e^x + 1 = 2$

47. $3e^x + 5 = 8$

48. $e^{2x} = 1$

49. $\ln x = 0$

50. $3 - \ln x = 3$

51. $\ln(3x) = 0$

52. $x^2 - 3x = 0$

53. $e^x + xe^x = 0$

54. $e^{2x} - e^x = 0$

Solve the following trig equations where $0 \leq x \leq 2\pi$.

55. $\sin x = \frac{1}{2}$

56. $\cos x = -1$

57. $\cos x = \frac{\sqrt{3}}{2}$

58. $2\sin x = -1$

59. $\cos x = \frac{\sqrt{2}}{2}$

60. $\cos\left(\frac{x}{2}\right) = \frac{\sqrt{3}}{2}$

61. $\tan x = 0$

62. $\sin(2x) = 1$

63. $\sin\left(\frac{x}{4}\right) = \frac{\sqrt{3}}{2}$

For each function, determine its domain and range.

Function	**Domain**	**Range**
64. $y = \sqrt{x-4}$		
65. $y = (x-3)^2$		
66. $y = \ln x$		
67. $y = e^x$		
68. $y = \sqrt{4 - x^2}$		

Simplify.

69. $\frac{\sqrt{x}}{x}$

70. $e^{\ln x}$

71. $e^{1+\ln x}$

72. $\ln 1$

73. $\ln e^7$

74. $\log_3 \frac{1}{3}$

75. $\log_{1/2} 8$

76. $\ln \frac{1}{2}$

77. $27^{\frac{2}{3}}$

78. $\left(5a^{2/3}\right)\left(4a^{3/2}\right)$

79. $\frac{4xy^{-2}}{12x^{-\frac{1}{3}}y^{-5}}$

80. $\left(4a^{5/3}\right)^{3/2}$

If $\boldsymbol{f(x) = \{(3,5),(2,4),(1,7)\}}$ $\quad$ $\boldsymbol{g(x) = \sqrt{x-3}}$
$\boldsymbol{h(x) = \{(3,2),(4,3),(1,6)\}}$ $\quad$ $\boldsymbol{k(x) = x^2 + 5}$ **, then determine each of the following.**

81. $(f+h)(1)$

82. $(k-g)(5)$

83. $f\left(h(3)\right)$

84. $g\left(k(7)\right)$

85. $h(3)$

86. $g\left(g(9)\right)$

87. $f^{-1}(4)$

88. $k^{-1}(x)$

89. $k\left(g(x)\right)$

90. $g(f(2))$

AP CALCULUS

CALCULATOR VIDEOS

https://calculus.flippedmath.com/02-calculator-skillz.html

Calculator Skillz

NOTES

CALCULUS

Write your questions here!

Calculator is always in RADIANS!

CALCULATOR TIPS

1. Use the calculator when it is faster
2. Justify your calculator work on Free Response
3. Do NOT round until the very end
4. Round/Truncate to 3 decimal places

$x = 43.58277289$

$x = -0.49927438$

Common Mistakes

Evaluate

$f(x) = x^2 - 4$ at $x = -3$

$f(x) = \frac{x}{2\pi}$ at $x = 7$

$f(x) = x^9 - 4$ at $x = 47$

Trig Functions

Evaluate

$f\left(\frac{\pi}{5}\right) = \csc\theta$

$f\left(\frac{2}{3\pi}\right) = \sin^2\theta$

Window, Trace, Table, ZStand and ZTrig

Evaluate

$f(x) = x^2 - 4$ at $x = 3.2$

$f(\theta) = \tan^{-1}(\theta)$ at $\theta = \pi$

ZFit, Finding Extrema and Roots

Find all Max/Min

$f(x) = x^4 - 3x^3 + x + 3$

Find the zeros

$f(x) = x^4 - 3x^3 + x + 3$

Finding Point of Intersection

Solve

$y = x^3 + 3x - 4$
$y = -x^2 + 4$

Solve Equations

Solve

$x^3 + 3x - 4 = 5$

STORE and RECALL

If $x = \sin\left(\frac{\pi}{7}\right)$, find $3^x - 2\sqrt{x} - 4x$

$f(x) = x^4 + 3x - 4$
$f(x) = -x^2 + 4$

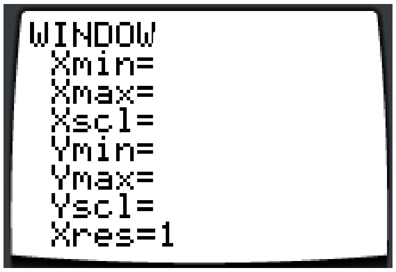
The x coordinate of the left point of intersection is A.
The x coordinate of the right point of intersection is B.

Find $A + B$

Window for Word Problems

Methane is produced in a cave at the rate of $r(t) = e^{\sin\left(\frac{\pi}{4}t\right)}$ liters per hour at time t hours. The initial amount of methane in the cave at time $t = 0$ is 20 liters. At $t = 8$ hours, a pump begins to remove the methane at a constant rate of 1.5 liters per hour.

At what time t during the time interval $0 \le t \le 8$ hours is the amount of methane increasing most rapidly?



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WINDOW
 Xmin=
 Xmax=
 Xscl=
 Ymin=
 Ymax=
 Yscl=
 Xres=1
```

SUMMARY:

Now, summarize your notes here!

You are allowed to use a graphing calculator for 1-21

Find all extrema and roots for each function.

1. $y = -\frac{9}{10}x^3 - \frac{3}{4}x^2 + 2x + 1$

Maximum Point(s) =

Minimum Point(s) =

Root(s) =

2. $f(x) = \frac{e^x - 1}{x^2 - 4}$

Maximum Point(s) =

Minimum Point(s) =

Root(s) =

Solve the systems of equations by graphing.

5. $y = -\ln(2x - 1) + 3$
$y = e^{\frac{2}{3}x} - 2$

6. $y = \sqrt{x^2 - 4}$
$y = \tan^{-1}(x) + 3$

Evaluate the function at the given point.

9. $f(x) = e^{x^2 - 1}$ at $x = e$

10. $y = \sec(x) + 5x$ at $x = \frac{\pi}{5}$

11. $f(x) = 3x\sqrt{x^2 + 5}$ at $x = \pi$

12. $y = 2\sin^2(x) + \tan(2x)$ at $x = \frac{\pi}{3}$

Use the STORE feature to evaluate the following.

13. STORE $x = \cot\left(\frac{\pi}{9}\right)$ and use RECALL to find

$$\sqrt{x} + \ln(2x) - e^x$$

14. STORE $x = e^{\pi}$ and use RECALL to find

$$4x - 2\sqrt{x^2 + 1} + 2^x$$

15. Solve the system of equations below. STORE the x coordinate of the left point of intersection as A. STORE the x coordinate of the right point of intersection as B.

$y = \sin^2(x^2) + 1$
$y = -|2x + 1| + 2.5$

Use RECALL to find $A - B$

16. STORE the x coordinate of the maximum point as A. STORE the x coordinate of the minimum point as B.

$y = -\frac{2}{5}x^3 - 2x^2 + x + 7$

Use RECALL to find $A - B$

State the WINDOW that allows you to view the function. Answer the question.

17. A tortoise runs along a straight track, starting at position $x = 0$ at time $t = 0$. The tortoise has a velocity of $v(t) = \ln(1 + t^2)$ inches per minute, where *t* is measured in minutes such that $0 \leq t \leq 15$.

What is the tortoise's velocity at $t = 2.5$?

WINDOW
Xmin=
Xmax=
Xscl=
Ymin=
Ymax=
Yscl=
Xres=1

18. A cylindrical can of radius 10 millimeters is used to measure rainfall in Stormville. The can is initially empty, and rain enters the can during a 30-day period. The rate at which the height of the water is rising in the can is given by $s(t) = 2\sin(0.03t) + 1.5$ where $s(t)$ is measured in millimeters per day and *t* is measured in days.

When will the rate of change of the height be 2 mm/day?

WINDOW
Xmin=
Xmax=
Xscl=
Ymin=
Ymax=
Yscl=
Xres=1

19. For $0 \leq t \leq 6$, a particle is moving along the *x*-axis. The particle's position, $x(t)$, is not explicitly given. The acceleration of the particle is given by $a(t) = \frac{1}{2}e^{t/4}\cos\left(e^{t/4}\right)$ in units per second^2.

(NOTE: Acceleration can be positive or negative!)

What is the particle's maximum acceleration?

WINDOW
Xmin=
Xmax=
Xscl=
Ymin=
Ymax=
Yscl=
Xres=1

20. The temperature on New Year's Day in Mathlandia was given by by $T(H) = -5 - 10\cos\left(\frac{\pi H}{12}\right)$ where *T* is the temperature in degrees Fahrenheit and *H* is the number of hours from midnight $0 \leq H \leq 24$.

Find $T(12)$ and explain what it means in this context.

WINDOW
Xmin=
Xmax=
Xscl=
Ymin=
Ymax=
Yscl=
Xres=1

21. A hospital patient is receiving a drug on an IV drip. The rate at which the drug enters the body is given by $E(t) = \frac{4}{1+e^{-t}}$ cubic centimeters per hour. The rate at which the body absorbs the drug is given by $D(t) = 3^{\sqrt{t}-1}$ cubic centimeters per hour. The IV drip starts at time $t = 0$ and continues for 8 hours until time $t = 8$.

Is the amount of drug in the body increasing or decreasing at $t = 6$?

WINDOW
Xmin=
Xmax=
Xscl=
Ymin=
Ymax=
Yscl=
Xres=1

You are allowed to use a graphing calculator for 1-4

MULTIPLE CHOICE

1. Find the value of x for which the graphs of $f(x) = \frac{1}{2}e^{x-4}$ and $g(x) = 3\sqrt[3]{x}$ have $f(x) = g(6)$.

   (A) -1.761
   (B) 0.35
   (C) 2.134
   (D) 5.451
   (E) 6.389

2. Find the minimum value of the function $f(x) = \ln(x) + \sin(x)$ on the interval $\left[\frac{\pi}{4}, \frac{9\pi}{4}\right]$.

   (A) 0.465
   (B) 0.526
   (C) 0.785
   (D) 1.145
   (E) 1.605

3. If $f(x) = -\frac{x^2}{x^3-8}$, how many values of c such satisfy the condition $f(c) = 0$?

   (A) 0
   (B) 1
   (C) 2
   (D) 3
   (E) 4

4. Which of the following statements about the function $y = x^3(3-x)$ is true?

   I. The function has an absolute maximum.
   II. The function has an absolute minimum.
   III. The function has a relative minimum.

   (A) I only
   (B) II only
   (C) III only
   (D) I and II
   (E) I and III

You are allowed to use a graphing calculator

FREE RESPONSE

Your score: ____ out of 5

An online retailer has a warehouse that receives packages that are later shipped out to customers. The warehouse is open 12 hours per day. On one particular day, packages are delivered to the warehouse at a rate of $D(t) = 300\sqrt{t} - 3t^2 + 75$ packages per hour. Packages are shipped out at a rate of $S(t) = 60t + 300\sin\left(\frac{\pi}{6}t\right) + 300$ packages per hour. For both functions, $0 \le t \le 12$, where t is measured in hours. At the beginning of the workday, the warehouse already has 4000 packages waiting to be shipped out.

1. What is the rate of change of the number of packages in the warehouse at time $t = 10$?

2. What is the rate of change of packages shipped out of the warehouse when the rate of change of packages delivered to the warehouse on this day is a maximum?

3. During what time interval(s) is the rate of packages being delivered to the warehouse greater than rate of packages being shipped out of the warehouse?

Calculus - SUMMER PACKET

NAME: Solutions

Summer + Math = (Best Summer Ever)^2

NO CALCULATOR!!!

Given $f(x) = x^2 - 2x + 5$, find the following.

1. $f(-2) =$

$f(-2) = (-2)^2 - 2(-2) + 5$

$f(-2) = 4 + 4 + 5$

$\boxed{f(-2) = 13}$

2. $f(x+2) =$

$\boxed{x^2 + 2x + 5}$

3. $f(x+h) =$

$(x+h)^2 - 2(x+h) + 5$

$(x+h)(x+h) - 2x - 2h + 5$

$x^2 + xh + xh + h^2 - 2x - 2h + 5$

$\boxed{x^2 + 2xh + h^2 - 2x - 2h + 5}$

Use the graph $f(x)$ to answer the following.

4. $f(0) = -4$

$f(-1) = -3.5$

$f(2) =$ DNE (Does not exist) or undefined

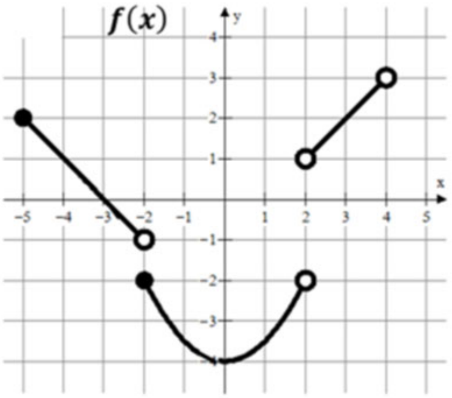
$f(x) = 2$ when $x = ?$ -5 and 3

$f(4) =$ DNE (Does not exist) or undefined

$f(-2) = -2$

$f(3) = 2$

$f(x) = -3$ when $x = ?$ -1.5 and 1.5



Write the equation of the line meets the following conditions. Use point-slope form.
$y - y_1 = m(x - x_1)$

5. slope = 3 and $(4, -2)$

$y - -2 = 3(x - 4)$

$\boxed{y + 2 = 3(x - 4)}$

6. $m = -\frac{3}{2}$ and $f(-5) = 7$

$\boxed{y - 7 = -\frac{3}{2}(x + 5)}$

7. $f(4) = -8$ and $f(-3) = 12$

$m = \frac{12 - -8}{-3 - 4} = \frac{20}{-7}$

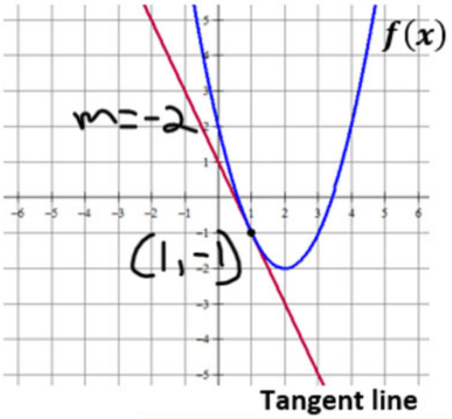
$\boxed{y - 12 = -\frac{20}{7}(x + 3)}$

or

$\boxed{y + 8 = -\frac{20}{7}(x - 4)}$

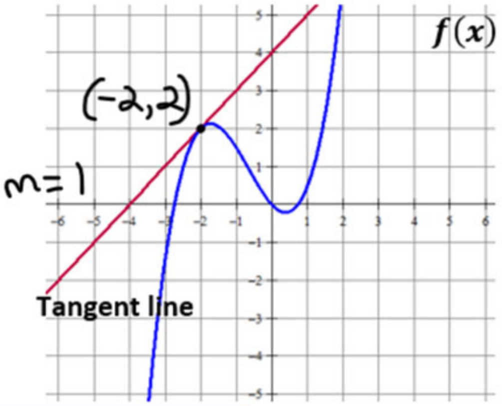
Write the equation of the tangent line in point slope form. $y - y_1 = m(x - x_1)$

8. The line tangent to $f(x)$ at $x = 1$



$\boxed{y+1=-2(x-1)}$

9. The line tangent to $f(x)$ at $x = -2$

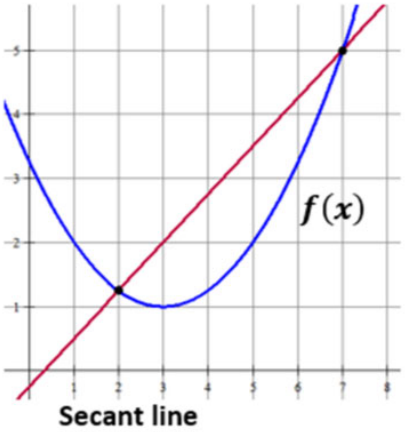


$\boxed{y-2=1(x+2)}$

MULTIPLE CHOICE! Remember slope = $\frac{y_2-y_1}{x_2-x_1}$

10. Which choice represents the slope of the secant line shown?

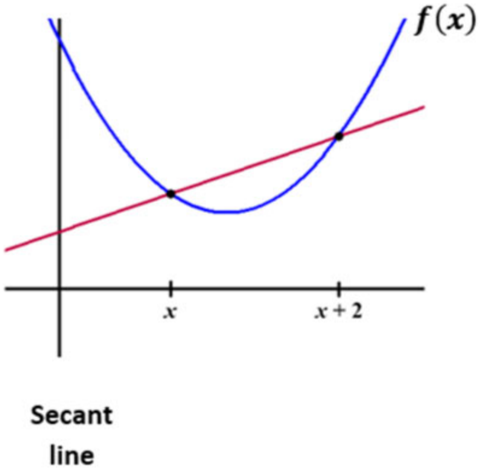
A) $\frac{7-2}{f(7)-f(2)}$ B) $\frac{f(7)-2}{7-f(2)}$ C) $\frac{7-f(2)}{f(7)-2}$ [✓ D) $\frac{f(7)-f(2)}{7-2}$]



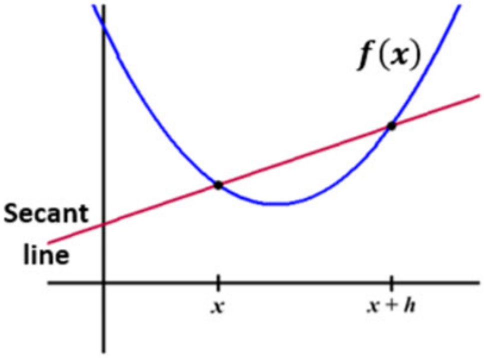
11. Which choice represents the slope of the secant line shown?

A) $\frac{f(x)-f(x+2)}{x+2-x}$ [✓ B) $\frac{f(x+2)-f(x)}{x+2-x}$] C) $\frac{f(x+2)-f(x)}{x-(x+2)}$

D) $\frac{x+2-x}{f(x)-f(x+2)}$



12. Which choice represents the slope of the secant line shown?



A) $\frac{f(x+h)-f(x)}{x-(x+h)}$

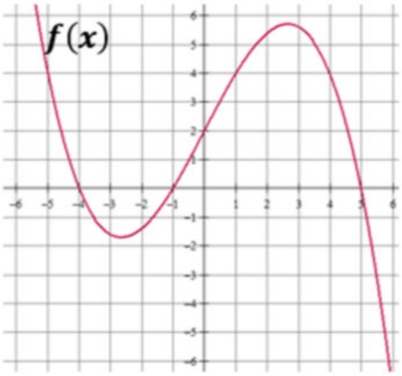
B) $\frac{x-(x+h)}{f(x+h)-f(x)}$

[✓ C) $\frac{f(x+h)-f(x)}{x+h-x}$]

D) $\frac{f(x)-f(x+h)}{x+h-x}$

13. Which of the following statements about the function $f(x)$ is true?

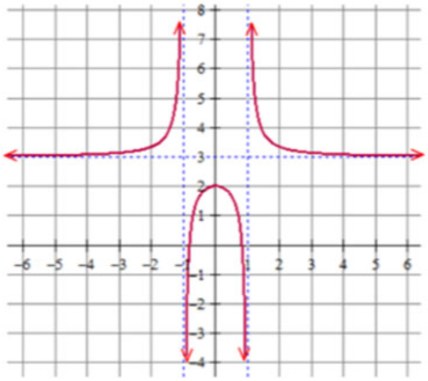
I. $f(2) = 0$
II. $(x + 4)$ is a factor of $f(x)$
III. $f(5) = f(-1)$



(A) I only
(B) II only
(C) III only
(D) I and III only
[✓ (E) II and III only]

Find the domain and range (express in interval notation). Find all horizontal and vertical asymptotes.

14.



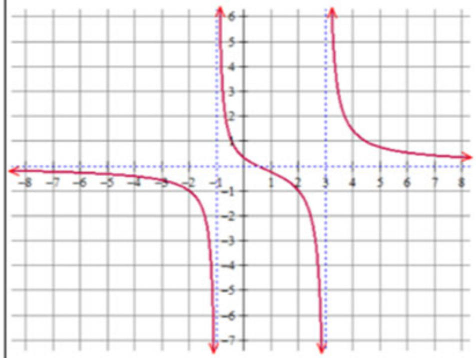
Domain: $(-\infty,-1)\cup(-1,1)\cup(1,\infty)$

Range: $(-\infty,2]\cup(3,\infty)$

Horizontal Asymptote(s): $y=3$

Vertical Asymptotes(s): $x=1$
$x=-1$

15.



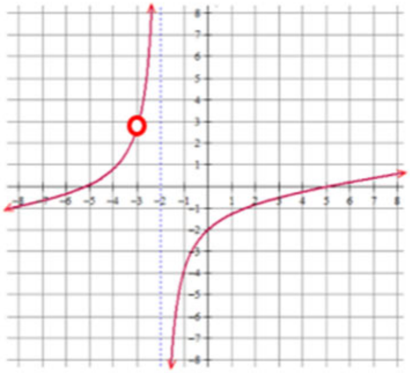
Domain: $(-\infty,-1)\cup(-1,3)\cup(3,\infty)$

Range: $(-\infty,\infty)$

Horizontal Asymptote(s): $y=0$

Vertical Asymptotes(s): $x=-1$
$x=3$

16.



Domain: $(-\infty,-3)\cup(-3,-2)\cup(-2,\infty)$

Range: $(-\infty,\infty)$

Horizontal Asymptote(s): none

Vertical Asymptotes(s): $x=-2$

MULTIPLE CHOICE!

17. Which of the following functions has a vertical asymptote at $= 4$?

(A) $\frac{x+5}{x^2-4}$

(B) $\frac{x^2-16}{x-4}$

(C) $\frac{4x}{x+1}$

[✓ (D) $\frac{x+6}{x^2-7x+12}$]

(E) None of the above

---

18. Consider the function: $(x) = \frac{x^2-5x+6}{x^2-4}$. Which of the following statements is true?

I. $f(x)$ has a vertical asymptote of $x = 2$
II. $f(x)$ has a vertical asymptote of $x = -2$
III. $f(x)$ has a horizontal asymptote of $y = 1$

(A) I only

(B) II only

(C) I and III only

[✓ (D) II and III only]

(E) I, II and III

Rewrite the following using rational exponents. Example: $\frac{1}{\sqrt[3]{x^2}} = x^{-\frac{2}{3}}$

19. $\sqrt[5]{x^3} + \sqrt[5]{2x}$ $x^{3/5} + (2x)^{1/5}$	20. $\sqrt{x+1}$ $(x+1)^{1/2}$	21. $\frac{1}{\sqrt{x+1}}$ $(x+1)^{-1/2}$
22. $\frac{1}{\sqrt{x}} - \frac{2}{x}$ $x^{-1/2} - 2x^{-1}$	23. $\frac{1}{4x^3} + \frac{1}{2}\sqrt[4]{x^3}$ $\frac{1}{4}x^{-3} + \frac{1}{2}x^{3/4}$	24. $\frac{1}{4\sqrt{x}} - 2\sqrt{x+1}$ $\frac{1}{4}x^{-1/2} - 2(x+1)^{1/2}$

Write each expression in radical form and positive exponents. Example: $x^{-\frac{2}{3}} + x^{-2} = \frac{1}{\sqrt[3]{x^2}} + \frac{1}{x^2}$

25. $x^{-\frac{1}{2}} - x^{\frac{3}{2}}$ $\frac{1}{\sqrt{x}} - \sqrt{x^3}$	26. $\frac{1}{2}x^{-\frac{1}{2}} + x^{-1}$ $\frac{1}{2\sqrt{x}} + \frac{1}{x}$	27. $3x^{-\frac{1}{2}}$ $\frac{3}{\sqrt{x}}$
28. $(x+4)^{-\frac{1}{2}}$ $\frac{1}{\sqrt{x+4}}$	29. $x^{-2} + x^{\frac{1}{2}}$ $\frac{1}{x^2} + \sqrt{x}$	30. $2x^{-2} + \frac{3}{2}x^{-1}$ $\frac{2}{x^2} + \frac{3}{2x}$

Need to know basic trig functions in RADIANS! We never use degrees. You can either use the Unit Circle or Special Triangles to find the following.

31. $\sin\frac{\pi}{6}$ $\frac{1}{2}$	32. $\cos\frac{\pi}{4}$ $\frac{\sqrt{2}}{2}$	33. $\sin 2\pi$ 0
34. $\tan\pi$ 0	35. $\sec\frac{\pi}{2}$ undefined	36. $\cos\frac{\pi}{6}$ $\frac{\sqrt{3}}{2}$
37. $\sin\frac{\pi}{3}$ $\frac{\sqrt{3}}{2}$	38. $\sin\frac{3\pi}{2}$ -1	39. $\tan\frac{\pi}{4}$ 1
40. $\csc\frac{\pi}{2}$ 1	41. $\sin\pi$ 0	42. $\cos\frac{\pi}{3}$ $\frac{1}{2}$
43. Find x where $0 \le x \le 2\pi$, $\sin x = \frac{1}{2}$ $\frac{\pi}{6}$ and $\frac{5\pi}{6}$	44. Find x where $0 \le x \le 2\pi$, $\tan x = 0$ 0, π, and 2π	45. Find x where $0 \le x \le 2\pi$, $\cos x = -1$ π

Solve the following equations. Remember $e^0 = 1$ and $\ln 1 = 0$.

46. $e^x + 1 = 2$ $e^x = 1$ $\ln(e^x) = \ln(1)$ $\boxed{x=0}$	47. $3e^x + 5 = 8$ $3e^x = 3$ $e^x = 1$ $\ln e^x = \ln 1$ $\boxed{x=0}$	48. $e^{2x} = 1$ $\ln e^{2x} = \ln(1)$ $2x = 0$ $\boxed{x=0}$
49. $\ln x = 0$ $e \quad e$ $\boxed{x=1}$	50. $3 - \ln x = 3$ $-\ln x = 0$ $\ln x = 0$ $e \quad e$ $\boxed{x=1}$	51. $\ln(3x) = 0$ $e \quad e$ $3x = 1$ $\boxed{x=\frac{1}{3}}$
52. $x^2 - 3x = 0$ $x(x-3) = 0$ $\boxed{x=0 \quad x=3}$	53. $e^x + xe^x = 0$ $e^x(1+x) = 0$ $e^x = 0$ $1+x=0$ not possible $\boxed{x=-1}$	54. $e^{2x} - e^x = 0$ $e^x(e^x - 1) = 0$ $e^x = 0$ $e^x - 1 = 0$ not possible $e^x = 1$ $\boxed{x=0}$

Solve the following trig equations where $0 \le x \le 2\pi$.

55. $\sin x = \frac{1}{2}$

$x=\frac{\pi}{6}$ and $\frac{5\pi}{6}$

56. $\cos x = -1$

$x=\pi$

57. $\cos x = \frac{\sqrt{3}}{2}$

$x=\frac{\pi}{6}$ and $\frac{11\pi}{6}$

58. $2\sin x = -1$

$\sin x = -\frac{1}{2}$

$\boxed{x=\frac{7\pi}{6} \text{ and } \frac{11\pi}{6}}$

59. $\cos x = \frac{\sqrt{2}}{2}$

$x=\frac{\pi}{4}$ and $\frac{7\pi}{4}$

60. $\cos\left(\frac{x}{2}\right) = \frac{\sqrt{3}}{2}$

$\frac{x}{2}=\frac{\pi}{6}$ $\quad$ $\frac{x}{2}=\frac{11\pi}{6}$

$\boxed{x=\frac{\pi}{3}}$ $\quad$ $x=\frac{11\pi}{3}$

not in the domain interval

61. $\tan x = 0$

$\frac{\sin x}{\cos x}=0 \rightarrow \sin x=0$

$\boxed{x=0, \pi, 2\pi}$

62. $\sin(2x) = 1$

$2x=\frac{\pi}{2}$ and $2x=\frac{5\pi}{2}$

$\boxed{x=\frac{\pi}{4} \text{ and } x=\frac{5\pi}{4}}$

63. $\sin\left(\frac{x}{4}\right) = \frac{\sqrt{3}}{2}$

$\frac{x}{4}=\frac{\pi}{3}$ $\quad$ $\frac{x}{4}=\frac{2\pi}{3}$

$\boxed{x=\frac{4\pi}{3} \text{ and } x=\frac{4\pi}{3}}$

For each function, determine its domain and range.

Function	**Domain**	**Range**
64. $y=\sqrt{x-4}$	$x\ge 4$	$y\ge 0$
65. $y=(x-3)^2$	$\mathbb{R}$ all real numbers	$y\ge 0$
66. $y=\ln x$	$x>0$	$\mathbb{R}$
67. $y=e^x$	$\mathbb{R}$	$y>0$
68. $y=\sqrt{4-x^2}$	$-2\le x\le 2$	$0\le y\le 2$

Simplify.

69. $\frac{\sqrt{x}}{x}$

$x^{\frac{1}{2}-1}$

$x^{-\frac{1}{2}}$

$\boxed{\frac{1}{\sqrt{x}}}$

70. $e^{\ln x}$

$\boxed{x}$

71. $e^{1+\ln x}$

$e^{1}\cdot e^{\ln x}$

$\boxed{ex}$

72. $\ln 1$

$\boxed{0}$

73. $\ln e^7$

$\boxed{7}$

74. $\log_3 \frac{1}{3}$

$\log_3 3^{-1}$

$\boxed{-1}$

75. $\log_{1/2} 8$

$\log_{\frac{1}{2}} (\frac{1}{2})^{-3}$

$\boxed{-3}$

76. $\ln \frac{1}{2}$ Calculator needed

$\approx \boxed{-0.693}$

77. $27^{\frac{2}{3}}$

$\sqrt[3]{27}^{\,2}$

3^2

$\boxed{9}$

78. $\left(5a^{2/3}\right)\left(4a^{3/2}\right)$

$20a^{\frac{2}{3}+\frac{3}{2}}$

$\boxed{20a^{\frac{13}{6}}}$

79. $\frac{4xy^{-2}}{12x^{-\frac{1}{3}}y^{-5}}$

$\frac{1}{3}x^{1--\frac{1}{3}}\ y^{-2--5}$

$\boxed{\frac{1}{3}x^{\frac{4}{3}}y^{3}}$

80. $\left(4a^{5/3}\right)^{3/2}$

$\sqrt{4}^{3}\ a^{\frac{5}{3}\cdot\frac{3}{2}}$

$\boxed{8a^{\frac{5}{2}}}$

If $f(x) = \{(3,5),(2,4),(1,7)\}$ $g(x) = \sqrt{x-3}$
$h(x) = \{(3,2),(4,3),(1,6)\}$ $k(x) = x^2+5$, then determine each of the following.

81. $(f+h)(1)$

$f(1)+h(1)$

$7 + 6$

$\boxed{13}$

82. $(k-g)(5)$

$k(5)-g(5)$

$(25+5)-(\sqrt{2})$

$\boxed{30-\sqrt{2}}$

83. $f(h(3))$

$f(2)$

$\boxed{4}$

84. $g(k(7))$

$g(7^2+5)$

$g(54)$

$\boxed{\sqrt{54-3} = \sqrt{51}}$

85. $h(3)$

$\boxed{2}$

86. $g(g(9))$

$g(\sqrt{6})$

$\boxed{\sqrt{\sqrt{6}-3}}$

87. $f^{-1}(4)$

$\boxed{2}$

88. $k^{-1}(x)$

$x = y^2+5$

$x-5 = y^2$

$\boxed{y = -\sqrt{x-5}}$

89. $k(g(x)) = (\sqrt{x-3})^2+5$

$x-3+5$

$\boxed{x+2}$

90. $g(f(2))$

$g(4) = \sqrt{4-3}$

$\boxed{1}$

You are allowed to use a graphing calculator for 1-21

Find all extrema and roots for each function.

1. $y = -\frac{9}{10}x^3 - \frac{3}{4}x^2 + 2x + 1$

Maximum Point(s) = $\mathbf{(0.626/7\,,1.737)}$

Minimum Point(s) = $\mathbf{(-1.182\,,-0.925/6)}$

Root(s) = $\mathbf{(-1.742/3\,,0), (-0.464\,,0)}$ **and** $\mathbf{(1.373/4\,,0)}$

2. $f(x) = \frac{e^x - 1}{x^2 - 4}$

Maximum Point(s) = **None**

Minimum Point(s) = $\mathbf{(3.175/6\,,3.77)}$

Root(s) = $\mathbf{(0,0)}$

Solve the systems of equations by graphing.

5. $y = -\ln(2x - 1) + 3$

$y = e^{\frac{2}{3}x} - 2$

$\mathbf{(2.033, 1.879)}$

6. $y = \sqrt{x^2 - 4}$

$y = \tan^{-1}(x) + 3$

$\mathbf{(-2.681, 1.786)}$ **and** $\mathbf{(4.801/2\,,4.365)}$

Evaluate the function at the given point.

9. $f(x) = e^{x^2 - 1}$ at $x = e$

$\mathbf{595.294}$

10. $y = \sec(x) + 5x$ at $x = \frac{\pi}{5}$

$\mathbf{4.377/8}$

11. $f(x) = 3x\sqrt{x^2 + 5}$ at $x = \pi$

$\mathbf{36.343}$

12. $y = 2\sin^2(x) + \tan(2x)$ at $x = \frac{\pi}{3}$

$\mathbf{-0.232}$

Use the STORE feature to evaluate the following.

13. STORE $x = \cot\left(\frac{\pi}{9}\right)$ and use RECALL to find

$\sqrt{x} + \ln(2x) - e^x$

$\mathbf{-12.241/2}$

14. STORE $x = e^{\pi}$ and use RECALL to find

$4x - 2\sqrt{x^2 + 1} + 2^x$

$\mathbf{9247935.122}$

15. Solve the system of equations below. STORE the *x* coordinate of the left point of intersection as A. STORE the *x* coordinate of the right point of intersection as B.

$y = \sin^2(x^2) + 1$

$y = -|2x + 1| + 2.5$

Use RECALL to find $A - B$

$\mathbf{-1.193/4}$

16. STORE the *x* coordinate of the maximum point as A. STORE the *x* coordinate of the minimum point as B.

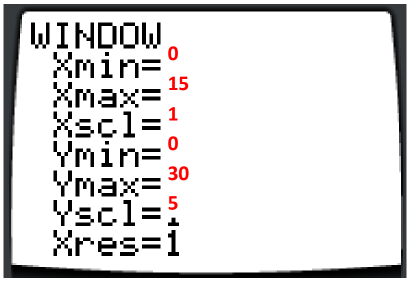
$y = -\frac{2}{5}x^3 - 2x^2 + x + 7$

Use RECALL to find $A - B$

$\mathbf{3.800/1}$

State the WINDOW that allows you to view the function. Answer the question.

17. A tortoise runs along a straight track, starting at position $x = 0$ at time $t = 0$. The tortoise has a velocity of $v(t) = \ln(1 + t^2)$ inches per minute, where t is measured in minutes such that $0 \le t \le 15$.



WINDOW
Xmin=0
Xmax=15
Xscl=1
Ymin=0
Ymax=30
Yscl=5
Xres=1

What is the tortoise's velocity at $t = 2.5$?

$\mathbf{1.981\ \textit{inches/min}}$

---

18. A cylindrical can of radius 10 millimeters is used to measure rainfall in Stormville. The can is initially empty, and rain enters the can during a 30-day period. The rate at which the height of the water is rising in the can is given by $s(t) = 2\sin(0.03t) + 1.5$ where $s(t)$ is measured in millimeters per day and t is measured in days.

WINDOW
Xmin=0
Xmax=30
Xscl=1
Ymin=0
Ymax=5
Yscl=1
Xres=1

When will the rate of change of the height be 2 mm/day?

On day $\mathbf{8.422/3}$

---

19. For $0 \le t \le 6$, a particle is moving along the x-axis. The particle's position, $x(t)$, is not explicitly given. The acceleration of the particle is given by $a(t) = \frac{1}{2}e^{t/4}\cos\left(e^{t/4}\right)$ in units per second^2.

(NOTE: Acceleration can be positive or negative!)

WINDOW
Xmin=0
Xmax=6
Xscl=1
Ymin=-2
Ymax=1
Yscl=0.5
Xres=1

What is the particle's maximum acceleration? $\mathbf{-1.644\ \textit{units/sec}^2}$

---

20. The temperature on New Year's Day in Mathlandia was given by by $T(H) = -5 - 10\cos\left(\frac{\pi H}{12}\right)$ where T is the temperature in degrees Fahrenheit and H is the number of hours from midnight $0 \le H \le 24$.

WINDOW
Xmin=0
Xmax=24
Xscl=1
Ymin=-20
Ymax=10
Yscl=5
Xres=1

Find $T(12)$ and explain what it means in this context.

$T(12) = 5$, this means that 12 hours after midnight (noon) the temperature on New Year's day in Mathlandia is 5°F

---

21. A hospital patient is receiving a drug on an IV drip. The rate at which the drug enters the body is given by $E(t) = \frac{4}{1+e^{-t}}$ cubic centimeters per hour. The rate at which the body absorbs the drug is given by $D(t) = 3^{\sqrt{t}-1}$ cubic centimeters per hour. The IV drip starts at time $t = 0$ and continues for 8 hours until time $t = 8$.

WINDOW
Xmin=0
Xmax=8
Xscl=1
Ymin=0
Ymax=10
Yscl=1
Xres=1

Is the amount of drug in the body increasing or decreasing at $t = 6$?

$\mathbf{E(6) - D(6) = -0.926\ cm^3/hr}$ **so decreasing!**

You are allowed to use a graphing calculator for 1-4

MULTIPLE CHOICE

1. **E**
2. **A**
3. **B**
4. **A**

FREE RESPONSE

Your score: ____ out of 5

An online retailer has a warehouse that receives packages that are later shipped out to customers. The warehouse is open 12 hours per day. On one particular day, packages are delivered to the warehouse at a rate of $D(t) = 300\sqrt{t} - 3t^2 + 75$ packages per hour. Packages are shipped out at a rate of $S(t) = 60t + 300\sin\left(\frac{\pi}{6}t\right) + 300$ packages per hour. For both functions, $0 \leq t \leq 12$, where t is measured in hours. At the beginning of the workday, the warehouse already has 4000 packages waiting to be shipped out.

1. What is the rate of change of the number of packages in the warehouse at time $t = 10$?

$$\boldsymbol{D(10) - S(10) = 83.491}\ \boldsymbol{\textit{packages per hour}}$$

1 point set up

1 point answer (must be labeled correctly)

2. What is the rate of change of packages shipped out of the warehouse when the rate of change of packages delivered to the warehouse on this day is a maximum?

$$\boldsymbol{S(8.549) = 521.286}\ \boldsymbol{\textit{packages shipped per hour}}$$

1 point x-value
at maximum

1 point answer (must be labeled correctly)

3. During what time interval(s) is the rate of packages being delivered to the warehouse greater than rate of packages being shipped out of the warehouse?

$$\boldsymbol{[5.660/1, 10.491/2]}$$

From 5.660/1 hours to 10.491/2

1 point for correct interval