Seckinger HS 2024 AP Calculus AB Summer Assignment

Welcome to AP Calculus AB. This assignment is meant to review **essential** prior knowledge as you head into AP Calculus AB. All of the material should look familiar to you from your previous classes. Additionally you should go through the **Getting Ready for AP Calculus** on Khan Academy at: https://www.khanacademy.org/math/get-ready-for-ap-calc . You will be responsible for all of the material in this packet and from Khan Academy as we will be having an assessment over it the first week of school. We will use the first 2 days of class to go over and answer any questions you may have on the assignment.

I am so excited about AP Calculus AB and I am looking forward to a great school year!

-Seckinger AP Calculus AB

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FUNCTIONS

To evaluate a function for a given value, simply plug the value into the function for x.

Recall: $(f \circ g)(x) = f(g(x))$ *OR* $f[g(x)]$ read "f of g of x" Means to plug the inside function (in this case g(x)) in for x in the outside function (in this case, f(x)).

Example: Given $f(x) = 2x^2 + 1$ *and* $g(x) = x - 4$ find *f(g(x))*.

$$\begin{aligned} f(g(x)) &= f(x-4) \\ &= 2(x-4)^2 + 1 \\ &= 2(x^2 - 8x + 16) + 1 \\ &= 2x^2 - 16x + 32 + 1 \\ f(g(x)) &= 2x^2 - 16x + 33 \end{aligned}$$

Let $f(x) = 2x + 1$ *and* $g(x) = 2x^2 - 1$. **Find each.**

1. $f(2) =$ ______________ 2. $g(-3) =$ ______________ 3. $f(t+1) =$ ____________

4. $f[g(-2)] =$ ____________ 5. $g[f(m+2)] =$ ____________ 6. $[f(x)]^2 - 2g(x) =$ _______

Let $f(x) = \sin(2x)$ **Find each exactly.**

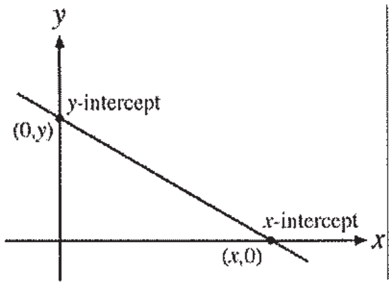
7. $f\left(\frac{\pi}{4}\right) =$ ____________ 8. $f\left(\frac{2\pi}{3}\right) =$ ________________

Let $f(x) = x^2$, $g(x) = 2x + 5$, *and* $h(x) = x^2 - 1$. **Find each.**

9. $h[f(-2)] =$ ________ 10. $f[g(x-1)] =$ ________ 11. $g\left[h(x^3)\right] =$ ________

INTERCEPTS OF A GRAPH

To find the x-intercepts, let y = 0 in your equation and solve.
To find the y-intercepts, let x = 0 in your equation and solve.



Example: Given the function $y = x^2 - 2x - 3$, find all intercepts.

$\underline{x - \text{int. } (Let\ y = 0)}$

$0 = x^2 - 2x - 3$

$0 = (x-3)(x+1)$

$x = -1 \ or \ x = 3$

$x -$ intercepts $(-1,0)$ *and* $(3,0)$

$\underline{y - \text{int. } (Let\ x = 0)}$

$y = 0^2 - 2(0) - 3$

$y = -3$

$y -$ intercept $(0,-3)$

Find the x and y intercepts for each.

12. $y = 2x - 5$

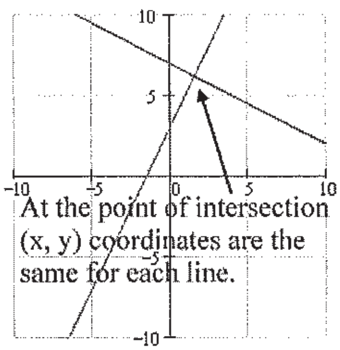
13. $y = x^2 + x - 2$

14. $y = x\sqrt{16 - x^2}$

15. $y^2 = x^3 - 4x$

POINTS OF INTERSECTION

Use substitution or elimination method to solve the system of equations.
Remember: You are finding a POINT OF INTERSECTION so your answer is an ordered pair.



> **CALCULATOR TIP**
> Remember you can use your calculator to verify your answers below. Graph the two lines then go to CALC (2^{nd} Trace) and hit INTERSECT.

Example: Find all points of intersection of $\begin{matrix} x^2 - y = 3 \\ x - y = 1 \end{matrix}$

ELIMINATION METHOD

Subtract to eliminate y

$x^2 - x = 2$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = 2$ or $x = -1$

Plug in $x = 2$ and $x = -1$ to find y

Points of Intersection: $(2,1)$ and $(-1,-2)$

SUBSTITUTION METHOD

Solve one equation for one variable.

$y = x^2 - 3$

$y = x - 1$

Therefore by substitution $x^2 - 3 = x - 1$

$x^2 - x - 2 = 0$

From here it is the same as the other example

Find the point(s) of intersection of the graphs for the given equations.

16. $\begin{matrix} x + y = 8 \\ 4x - y = 7 \end{matrix}$

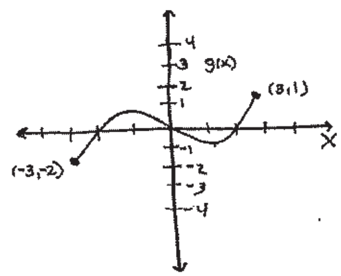
17. $\begin{matrix} x^2 + y = 6 \\ x + y = 4 \end{matrix}$

18. $\begin{matrix} x = 3 - y^2 \\ y = x - 1 \end{matrix}$

DOMAIN AND RANGE

Domain – All x values for which a function is defined (input values)
Range – Possible y or Output values

EXAMPLE 1



a) Find Domain & Range of g(x).
The domain is the set of inputs ~~out~~ of the function.
Input values run along the horizontal axis.
The furthest left input value associated with a pt. on the graph is -3. The furthest right input values associated with a pt. on the graph is 3.
So Domain is [-3,3], that is all reals from -3 to 3.

The range represents the set of output values for the function. Output values run along the vertical axis.
The lowest output value of the function is -2. The highest is 1. So the range is [-2,1], all reals from -2 to 1.

EXAMPLE 2

Find the domain and range of $f(x)=\sqrt{4-x^2}$
Write answers in interval notation.

DOMAIN
For $f(x)$ to be defined $4-x^2 \geq 0$.
This is true when $-2 \leq x \leq 2$
Domain: $[-2,2]$

RANGE
The solution to a square root must always be positive thus $f(x)$ must be greater than or equal to 0.
Range: $[0,\infty)$

Find the domain and range of each function. Write your answer in INTERVAL notation.

19. $f(x)=x^2-5$

20. $f(x)=-\sqrt{x+3}$

21. $f(x)=3\sin x$

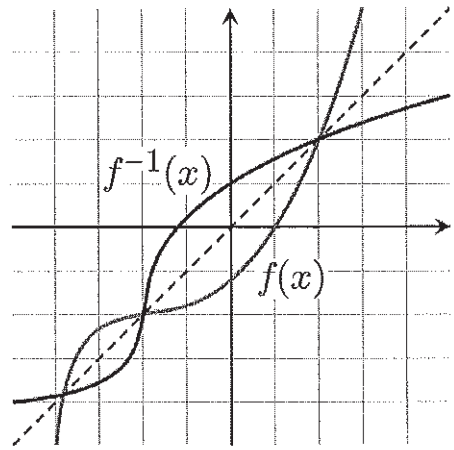
22. $f(x)=\dfrac{2}{x-1}$

INVERSES

To find the inverse of a function, simply switch the x and the y and solve for the new "y" value. Recall $f^{-1}(x)$ is defined as the inverse of $f(x)$

Example 1:

$f(x)=\sqrt[3]{x+1}$	Rewrite f(x) as y
$y=\sqrt[3]{x+1}$	Switch x and y
$x=\sqrt[3]{y+1}$	Solve for your new y
$(x)^3=\left(\sqrt[3]{y+1}\right)^3$	Cube both sides
$x^3=y+1$	Simplify
$y=x^3-1$	Solve for y
$f^{-1}(x)=x^3-1$	Rewrite in inverse notation



Find the inverse for each function.

23. $f(x)=2x+1$

24. $f(x)=\dfrac{x^2}{3}$

25. $g(x)=\dfrac{5}{x-2}$

26. $y=\sqrt{4-x}+1$

27. If the graph of $f(x)$ has the point (2, 7) then what is one point that will be on the graph of $f^{-1}(x)$?

28. Explain how the graphs of $f(x)$ and $f^{-1}(x)$ compare.

EQUATION OF A LINE

Slope intercept form: $y = mx + b$

Vertical line: x = c (slope is undefined)

Point-slope form: $y - y_1 = m(x - x_1)$
* LEARN! We will use this formula frequently!

Horizontal line: y = c (slope is 0)

Example: Write a linear equation that has a slope of ½ and passes through the point (2, -6)

Slope intercept form

$y = \frac{1}{2}x + b$	Plug in ½ for m
$-6 = \frac{1}{2}(2) + b$	Plug in the given ordered
$b = -7$	Solve for b
$y = \frac{1}{2}x - 7$	

Point-slope form

$y + 6 = \frac{1}{2}(x - 2)$	Plug in all variables
$y = \frac{1}{2}x - 7$	Solve for y

29. Determine the equation of a line passing through the point (5, -3) with an undefined slope.

30. Determine the equation of a line passing through the point (-4, 2) with a slope of 0.

31. Use point-slope form to find the equation of the line passing through the point (0, 5) with a slope of 2/3.

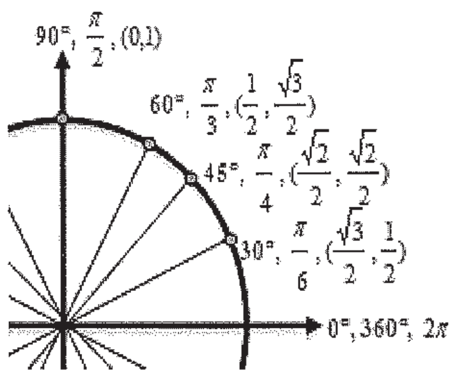
32. Use point-slope form to find a line passing through the point (2, 8) and parallel to the line $y = \frac{5}{6}x - 1$.

33. Use point-slope form to find a line perpendicular to $y = -2x + 9$ passing through the point (4, 7).

34. Find the equation of a line passing through the points (-3, 6) and (1, 2).

35. Find the equation of a line with an x-intercept (2, 0) and a y-intercept (0, 3)

UNIT CIRCLE



You can determine the sine or the cosine of any standard angle on the unit circle. The x-coordinate of the circle is the cosine and the y-coordinate is the sine of the angle. Recall tangent is defined as sin/cos or the slope of the line.

Examples:

$\sin\frac{\pi}{2} = 1$ $\quad\cos\frac{\pi}{2} = 0$ $\quad\tan\frac{\pi}{2} = und$

***You must have these memorized OR know how to calculate their values without the use of a calculator.**

36. *a.)* $\sin\pi$ $\quad$ *b.)* $\cos\frac{3\pi}{2}$ $\quad$ *c.)* $\sin\left(-\frac{\pi}{2}\right)$ $\quad$ *d.)* $\sin\left(\frac{5\pi}{4}\right)$

e.) $\cos\frac{\pi}{4}$ $\quad$ *f.)* $\cos(-\pi)$ $\quad$ g) $\cos\frac{\pi}{3}$ $\quad$ h) $\sin\frac{5\pi}{6}$

i) $\cos\frac{2\pi}{3}$ $\quad$ j) $\tan\frac{\pi}{4}$ $\quad$ k) $\tan\pi$ $\quad$ l) $\tan\frac{\pi}{3}$

m) $\cos\frac{4\pi}{3}$ $\quad$ n) $\sin\frac{11\pi}{6}$ $\quad$ o) $\tan\frac{7\pi}{4}$ $\quad$ p) $\sin\left(-\frac{\pi}{6}\right)$

TRIGONOMETRIC EQUATIONS

Solve each of the equations for $0 \le x < 2\pi$.

37. $\sin x = -\frac{1}{2}$

38. $2\cos x = \sqrt{3}$

39. $4\sin^2 x = 3$
    **Recall $\sin^2 x = (\sin x)^2$
    **Recall if $x^2 = 25$ then $x = \pm 5$

40. $2\cos^2 x - 1 - \cos x = 0$ *Factor

TRANSFORMATION OF FUNCTIONS

$h(x) = f(x) + c$	Vertical shift c units up	$h(x) = f(x - c)$	Horizontal shift c units right
$h(x) = f(x) - c$	Vertical shift c units down	$h(x) = f(x + c)$	Horizontal shift c units left
$h(x) = -f(x)$	Reflection over the x-axis		

41. Given $f(x) = x^2$ and $g(x) = (x-3)^2 + 1$. How the does the graph of g(x) differ from f(x)?

42. Write an equation for the function that has the shape of $f(x) = x^3$ but moved six units to the left and reflected over the x-axis.

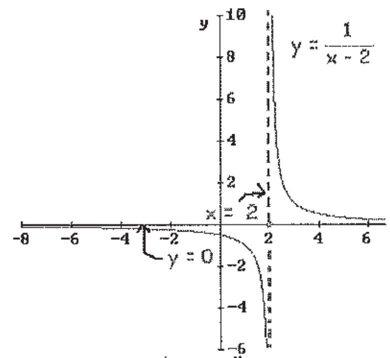
43. If the ordered pair (2, 4) is on the graph of $f(x)$, find one ordered pair that will be on the following functions:

a) $f(x) - 3$ b) $f(x - 3)$ c) $2f(x)$ d) $f(x - 2) + 1$ e) $-f(x)$

VERTICAL ASYMPTOTES

Determine the vertical asymptotes for the function. Set the denominator equal to zero to find the x-value for which the function is undefined. That will be the vertical asymptote given the numerator does not equal 0 also (Remember this is called removable discontinuity).
Write a vertical asymptotes as a line in the form x =

Example: Find the vertical asymptote of $y = \dfrac{1}{x-2}$
Since when $x = 2$ the function is in the form 1/0 then the vertical line $x = 2$ is a vertical asymptote of the function.



44. $f(x) = \dfrac{1}{x^2}$

45. $f(x) = \dfrac{x^2}{x^2 - 4}$

46. $f(x) = \dfrac{2 + x}{x^2(1 - x)}$

47. $f(x) = \dfrac{4 - x}{x^2 - 16}$

48. $f(x) = \dfrac{x - 1}{x^2 + x - 2}$

49. $f(x) = \dfrac{5x + 20}{x^2 - 16}$

HORIZONTAL ASYMPTOTES

Determine the horizontal asymptotes using the three cases below.

Case I. Degree of the numerator is less than the degree of the denominator. The asymptote is y = 0.

Example: $y = \dfrac{1}{x-1}$ (As x becomes very large or very negative the value of this function will approach 0). Thus there is a horizontal asymptote at $y = 0$.

Case II. Degree of the numerator is the same as the degree of the denominator. The asymptote is the ratio of the lead coefficients.

Exmaple: $y = \dfrac{2x^2+x-1}{3x^2+4}$ (As x becomes very large or very negative the value of this function will approach 2/3). Thus there is a horizontal asymptote at $y = \dfrac{2}{3}$.

Case III. Degree of the numerator is greater than the degree of the denominator. There is no horizontal asymptote. The function increases without bound. (If the degree of the numerator is exactly 1 more than the degree of the denominator, then there exists a slant asymptote, which is determined by long division.)

Example: $y = \dfrac{2x^2+x-1}{3x-3}$ (As x becomes very large the value of the function will continue to increase and as x becomes very negative the value of the function will also become more negative).

Determine all Horizontal Asymptotes.

50. $f(x) = \dfrac{x^2-2x+1}{x^3+x-7}$

51. $f(x) = \dfrac{5x^3-2x^2+8}{4x-3x^3+5}$

52. $f(x) = \dfrac{4x^2}{3x^2-7}$

53. $f(x) = \dfrac{(2x-5)^2}{x^2-x}$

54. $f(x) = \dfrac{-3x+1}{\sqrt{x^2+x}}$ * Remember $\sqrt{x^2} = \pm x$

This is very important in the use of limits.

EXPONENTIAL FUNCTIONS

Example: Solve for x

$4^{x+1} = \left(\frac{1}{2}\right)^{3x-2}$

$\left(2^2\right)^{x+1} = \left(2^{-1}\right)^{3x-2}$ Get a common base

$2^{2x+2} = 2^{-3x+2}$ Simplify

$2x+2 = -3x+2$ Set exponents equal

$x = 0$ Solve for x

Solve for x:

55. $3^{3x+5} = 9^{2x+1}$

56. $\left(\frac{1}{9}\right)^{x} = 27^{2x+4}$

57. $\left(\frac{1}{6}\right)^{x} = 216$

LOGARITHMS

The statement $y = b^x$ can be written as $x = \log_b y$. They mean the same thing.

REMEMBER: A LOGARITHM IS AN EXPONENT

Recall $\ln x = \log_e x$

The value of e is 2.718281828... or $\lim\limits_{x\to\infty}\left(1+\frac{1}{x}\right)^{x}$

Example: Evaluate the following logarithms

$\log_2 8 = ?$

In exponential for this is $2^{?} = 8$

Therefore $? = 3$

Thus $\log_2 8 = 3$

Evaluate the following logarithms

58. $\log_7 7$

59. $\log_3 27$

60. $\log_2 \frac{1}{32}$

61. $\log_{25} 5$

62. $\log_9 1$

63. $\log_4 8$

64. $\ln\sqrt{e}$

65. $\ln\frac{1}{e}$

PROPERTIES OF LOGARITHMS

$\log_b xy = \log_b x + \log_b y$ $\quad$ $\log_b \frac{x}{y} = \log_b x - \log_b y$ $\quad$ $\log_b x^y = y \log_b x$ $\quad$ $b^{\log_b x} = x$

Examples:

Expand $\log_4 16x$	Condense $\ln y - 2\ln R$	Expand $\log_2 7x^5$
$\log_4 16 + \log_4 x$	$\ln y - \ln R^2$	$\log_2 7 + \log_2 x^5$
$2 + \log_4 x$	$\ln \frac{y}{R^2}$	$\log_2 7 + 5\log_2 x$

Use the properties of logarithms to evaluate the following

66. $\log_2 2^5$

67. $\ln e^3$

68. $\log_2 8^3$

69. $\log_3 \sqrt[5]{9}$

70. $2^{\log_2 10}$

71. $e^{\ln 8}$

72. $9 \ln e^2$

73. $\log_9 9^3$

74. $\log_{10} 25 + \log_{10} 4$

75. $\log_2 40 - \log_2 5$

76. $\log_2 \left(\sqrt{2}\right)^5$

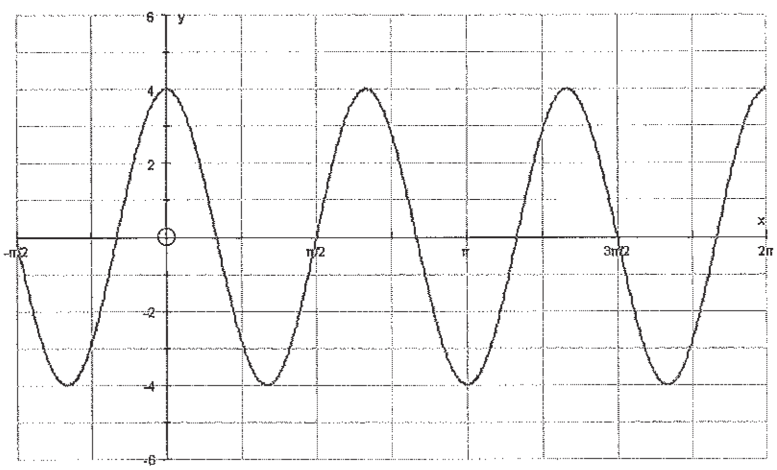
EVEN AND ODD FUNCTIONS

Recall:
Even functions *are functions that are symmetric over the y-axis.*
To determine algebraically we find out if $f(x) = f(-x)$
*(*Think about it what happens to the coordinate* $(x, f(x))$ *when reflected across the y-axis*)*

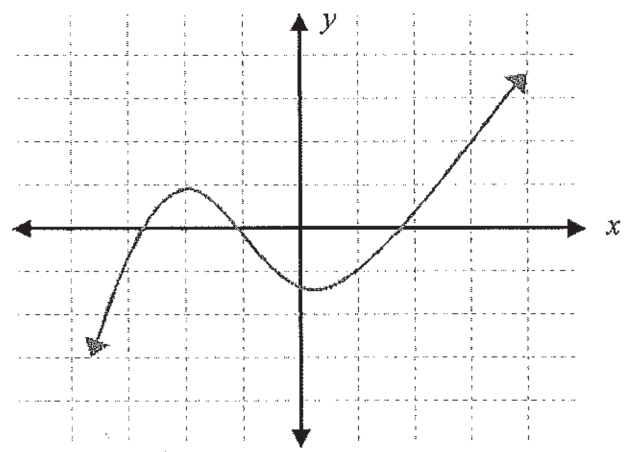
Odd functions *are functions that are symmetric about the origin.*
To determind algebraically we find out if $f(-x) = -f(x)$
*(*Think about it what happens to the coordinate* $(x, f(x))$ *when reflected over the origin*)*

State whether the following graphs are even, odd or neither, show ALL work.

77. ____________________



78. ____________________



79. ____________________
$f(x) = 2x^4 - 5x^2$

80. ____________________
$g(x) = x^5 - 3x^3 + x$

81. ____________________
$h(x) = 2x^2 - 5x + 3$

82. ____________________
$j(x) = 2\cos x$

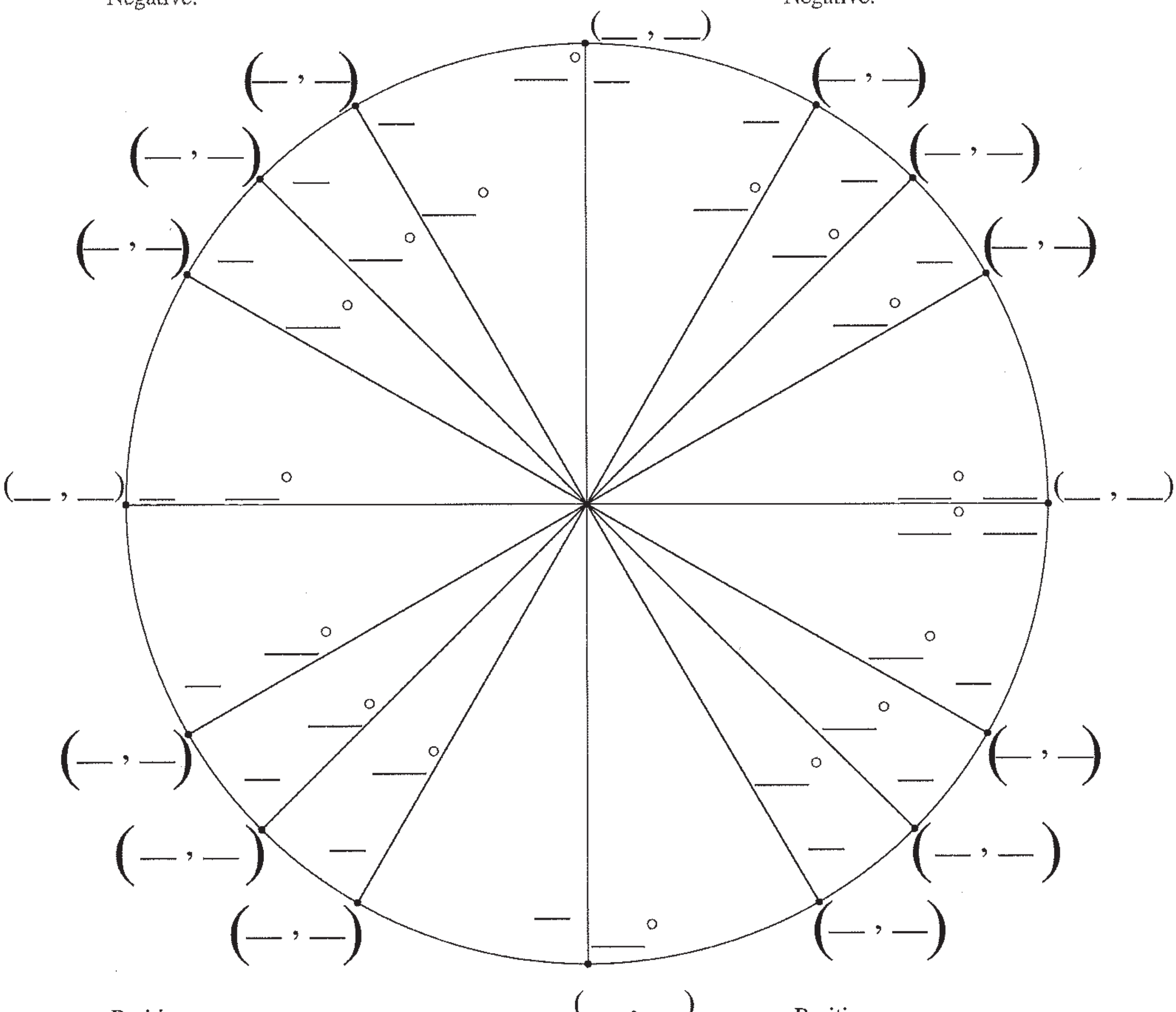
83. ____________________
$k(x) = \sin x + 4$

84. ____________________
$l(x) = \cos x - 3$

Fill in The Unit Circle

Positive:
Negative:

Positive:
Negative:



Positive:
Negative:

Positive:
Negative:

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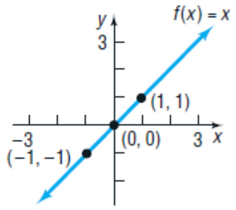
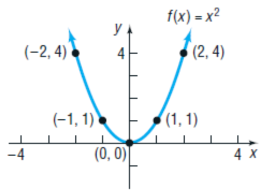
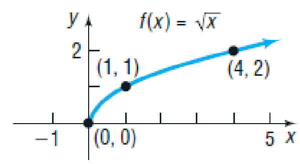
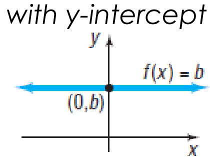
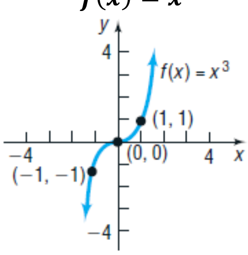
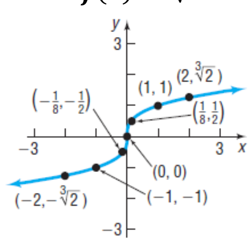
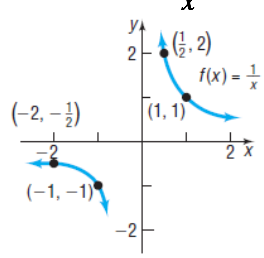
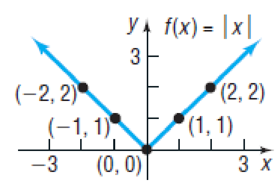
Families of Functions and their characteristics – A review

For each of the parent functions below identify domain, range, x-intercept(s), y-intercept, interval of increase, interval of decrease, intervals where $f(x) > 0\ (positive)$ and $f(x) < 0\ (negative)$, vertical asymptote $(x = \#)$ (if any), horizontal asymptote $(y = \#)$ (if any)

Identity Function $f(x) = x$	**Square/Quadratic Function** $f(x) = x^2$	**Square Root Function** $f(x) = \sqrt{x}$	**Constant Function** $f(x) = b$ | $f(x) = \#$ *Graph is a horizontal line with y-intercept* ***b.***
			
Domain: Range: x-intercept(s): y-intercept: Interval of Increase: Interval of Decrease: Interval where $f(x) > 0$ Interval where $f(x) < 0$ Vertical Asymptote Horizontal Asymptote	Domain: Range: x-intercept(s): y-intercept: Interval of Increase: Interval of Decrease: Interval where $f(x) > 0$ Interval where $f(x) < 0$ Vertical Asymptote Horizontal Asymptote	Domain: Range: x-intercept(s): y-intercept: Interval of Increase: Interval of Decrease: Interval where $f(x) > 0$ Interval where $f(x) < 0$ Vertical Asymptote Horizontal Asymptote	Domain: Range: x-intercept(s): y-intercept: Interval of Increase: Interval of Decrease: Interval where $f(x) > 0$ Interval where $f(x) < 0$ Vertical Asymptote Horizontal Asymptote

Cube Function $f(x) = x^3$	**Cube Root Function** $f(x) = \sqrt[3]{x}$	**Reciprocal Function** $f(x) = \dfrac{1}{x}$	**Absolute Value Function** $f(x) = \lvert x \rvert$
			
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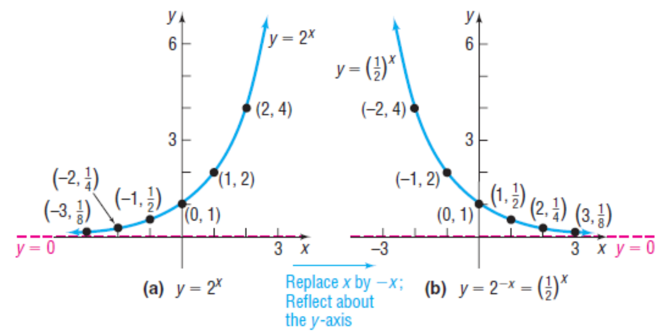
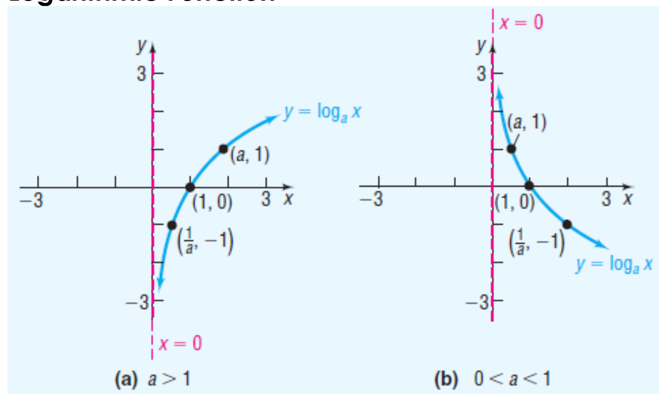
Exponential Function	Logarithmic Function
	

For $f(x) = 2^x$	For $f(x) = \frac{1^x}{2}$	For $f(x) = \log_a x$, $a > 1$	$f(x) = \log_a x$, $0 < a < 1$
Domain:	Domain:	Domain:	Domain:
Range:	Range:	Range:	Range:
x-intercept(s):	x-intercept(s):	x-intercept(s):	x-intercept(s):
y-intercept:	y-intercept:	y-intercept:	y-intercept:
Interval of Increase:	Interval of Increase:	Interval of Increase:	Interval of Increase:
Interval of Decrease:	Interval of Decrease:	Interval of Decrease:	Interval of Decrease:
Interval where $f(x) > 0$	Interval where $f(x) > 0$	Interval where $f(x) > 0$	Interval where $f(x) > 0$
Interval where $f(x) < 0$	Interval where $f(x) < 0$	Interval where $f(x) < 0$	Interval where $f(x) < 0$
Vertical Asymptote	Vertical Asymptote	Vertical Asymptote	Vertical Asymptote
Horizontal Asymptote	Horizontal Asymptote	Horizontal Asymptote	Horizontal Asymptote

Working with INTERVAL NOTATION

Interval Notation

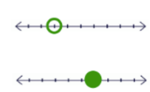
- gives the end points (beginning value and ending value) of a solution.

- if a value is included we use either open brackets [or closed brackets]

- if a value is excluded we use either open parenthesis (or closed parenthesis).

- infinity can never be included so we always use parentheses for infinity.

SYMBOL	MEANING
<	less than
>	greater than
≤	less than or equal to
≥	greater than or equal to



< , > : excluded ( )

≤ , ≥ : included [ ]

For $-\infty$ and ∞ always use parentheses ( )

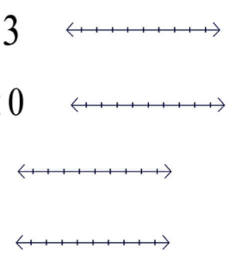
Graph then write each inequality using interval notation.

1) $1 \le x \le 3$

2) $-4 < x < 0$

3) $x > 5$

4) $x \le 1$



Set Notation

We can express solutions of inequalities in:

SET BUILDER NOTATION

$\{x \mid x \in \mathbb{R}, \textit{inequality solution}\}$

the set of all x such that x belongs to the real numbers system **and inequality solution**

Or: $\{x \mid x \in \mathbb{R}, solution/interval\}$